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ON MECHANISM OF DROPWISE CONDENSATION

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Abstract—The effect of surface thermal properties on dropwise condensation is considered. The physical explanation for this effect is given by considering constriction phenomena in metalic surfaces caused by a non-uniform heat flux distribution over the condensing surfaces. From available experimental data it is shown that surface thermal properties are one of the controlling factors in the dropwise condensation. It is found that for stainless steel as the condensing surface about 84 per cent of the total resistance was the contribution of the resistance occurring in the plate itself due to heat flux non-uniformity over the surface: for copper this contribution was 20 per cent.

NOMENCLATURE

A,	area;
b,	defined in Fig. 3;
c,	radius of an inactive drop;
h,	heat-transfer coefficient;
h_e,	liquid vapor heat-transfer coefficient;
h_{fg},	latent heat of vaporization;
g,	acceleration of gravity;
J_n,	Bessel function of first kind;
k_l,	thermal conductivity of liquid;
k_s,	thermal conductivity of solid;
n,	number of inactive drops per unit area;
p_{sat},	saturated pressure corresponding to T_v;
Q,	rate of heat transfer;
Q_d,	rate of heat transfer through a drop;
R,	Gas constant; local surface-to-liquid resistance;
R_0,	thermal resistance in dropwise condensation excluding constriction resistance in the surface;
R_s,	constriction resistance;
r,	drop radius, coordinate axis in the Appendix A;
r^*,	critical drop radius (the minimum size drop which can exist on the surface for given condition);
T_c,	temperature of the surface below an active drop;
T_s,	average surface temperature;
T_v,	vapor temperature;
z,	coordinate axis.

Greek symbols

α_n,	eiquevalues;
β,	fraction of the area covered by visible drops;
γ,	"condensation coefficient";
ΔT,	effective vapor-to-surface temperature difference;
ε,	fractional active area;
ψ,	constriction resistance factor.

1. INTRODUCTION

THE MECHANISM of dropwise condensation was considered during the last few decades by different authors. Jacob [1] and later others [2–5] attributed high heat transfer to a presence of a thin layer. McCormick and Baer [6] and Umur and Griffith [7] arrived at a conclusion that during dropwise condensation, condensation actually takes place on surfaces of large number of drops of different size. Moreover, it was concluded in [7] that condensate films greater than one-molecular thickness do not form on the surface. McCormick and Westwater [8, 9], presented important experimental observations elucidating the complex nature of dropwise condensation, in particular, they gave a detailed account of nucleation phenomena in [8] and active sites population, drop size distribution, coalescence and drop growth in [9]. Le Fevre and Rose [10] and Rose [11] had proposed overall model which, in addition to conduction through drops, takes into account surface tension effect, interface matter-transfer pressure drop, droplet size distribution and falling drops effect (the latter in [11] only).

However, none of the proposed theories have explicitly considered the effect of thermal properties of the surface material in spite of the fact that strong experimental evidence have existed showing that this effect is significant. First evidence was given in the work of Tanner *et al.* [12], where it was observed that heat-transfer coefficient was, in average, about five times less in the case of stainless steel than in the case of copper as a material for the condensing surface. They concluded that reduction in heat-transfer coefficient was not associated with any difference in the appearance of the condensing surface, and suggested that such a difference could arise from the influence of absorption properties and surface free energy of the interface.

Griffith and Lee [13] conclusively showed that the above explanation for the big difference in the heat-transfer coefficient for the two metals is invalid. They had their surfaces prepared identically and gold plated, and the difference in heat-transfer coefficients was still present in approximately the same degree. Their conclusion was that non-uniformity in surface temperature under conditions of dropwise condensation causes different heat fluxes for different surface materials and that higher thermal conductivity of the surface would give more uniform surface temperature and therefore higher heat flux (all for a fixed temperature difference between condensing vapor and the mean surface temperature). The argument given above is correct as it will be shown later.

The heat-transfer coefficient is defined as:

$$h = \frac{Q/A}{T_v - T_s} \tag{1}$$

where T_s is a hypothetical temperature of the condensing surface, obtained for a steady state, in a manner shown in Fig. 1. (T_s would be the true surface temperature if the surface material had an infinite conductivity in the lateral direction.) T_v is the vapor temperature; Q/A is the heat flux per unit area of the condensing surface.

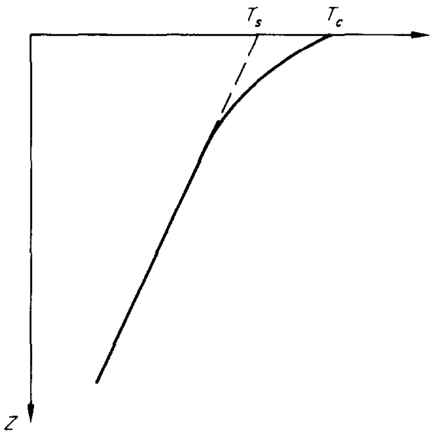


FIG. 1. Local heat flux influence on the local surface temperature.

The purpose of this work is to show that h, as defined by equation (1), depends not only on the surface-to-vapor resistance but also on the degree of surface temperature non-uniformity. The latter can be explicitly related to the thermal conductivity of the surface material and the distribution of drops over the surface.

In the first part of the work the total resistance occuring in the dropwise condensation was expressed as a sum of two resistances in series, namely, surface-to-vapor resistance and constriction resistance caused by temperature non-uniformity on the condensing surface. An expression for the constriction resistance, based on a model adopted here, is developed. From the published experimental data it is shown that the constriction resistance is a significant factor in dropwise condensation.

In the second part the constriction resistance was incorporated to an expression for an overall heat-transfer coefficient. This expression assumes the knowledge of the droplets distribution on the condensing surface.

2. CONSTRICTION RESISTANCE IN DROPWISE CONDENSATION

2.1 *Mechanism of the constriction resistance*

Local surface-to-vapor resistance (R) which reflects local conditions over the condensing surface, is highly non-uniform in dropwise condensation. Consequently, the heat flux distribution over the surface is also non-uniform. It further follows from the simple argument presented in Fig. 1 that for any surface with a finite thermal conductivity, the local temperature (T_c) is dependent on the local heat flux. For a given heat flux Q/A (average for the whole condensing surface), i.e. fixed T_s a lower R produces higher local heat flux, the latter in turn, implies higher T_c.

The heat flux over condensing surface is given by:

$$\frac{Q}{A} = \frac{1}{A}\int_A \frac{T_v - T_c}{R}\,\mathrm{d}A. \tag{2}$$

For a hypothetical surface material with the infinite thermal conductivity in the lateral direction, we would have $T_c = T_s$ and:

$$\frac{Q}{A} = \frac{T_v - T_s}{R_0}$$

where:

$$\frac{1}{R_0} \equiv \frac{1}{A}\int_A \frac{1}{R}\,\mathrm{d}A. \tag{3}$$

For a real surface, where T_c is variable over the surface, by defining T_{cm}, a weighted mean surface temperature with weighting function R_0/R as:

$$T_{cm} \equiv \frac{R_0}{A}\int_A \frac{T_c}{R}\,\mathrm{d}A, \tag{4}$$

we can present expression (2) in the form:

$$\frac{Q}{A} = \frac{T_v - T_{cm}}{R_0} = \frac{T_v - T_s}{R_0}\left(\frac{T_v - T_{cm}}{T_v - T_s}\right). \tag{5}$$

From our previous discussions concerning dependence between local temperature T_c and the local R, and the definition of T_{cm}, equation (4), it follows that $T_{cm} > T_s$. Hence, one can conclude from relation (5) that for a potential drop $(T_v - T_s)$ used in the definition of condensation heat transfer coefficient, equation (1), R_0 is not the only resistance in the system. Equivalently, droplets distribution on the condensing surface, the amount of non-condensible gases present in the vapor and all other system variables affecting R_0 are not sufficient to predict h. If the dropwise condensation process is to be considered as the heat flow due to the potential difference $(T_v - T_s)$ then an additional resistance should be introduced into the system in the amount of:

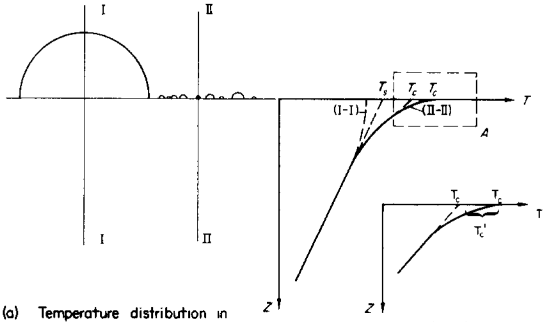
$$R_s \equiv \frac{T_{cm} - T_s}{Q/A}. \tag{6}$$

The above resistance is caused by constriction of heat flow lines in the plate near the condensing surface. One can now write for the heat-transfer coefficient from equations (1), (5) and (6) the following:

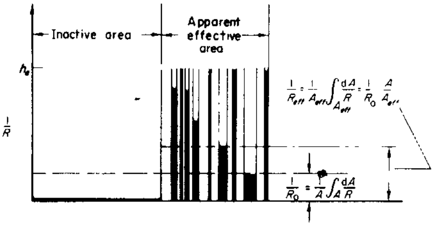
$$h^{-1} = \frac{T_v - T_s}{Q/A} = \frac{T_v - T_{cm}}{Q/A} + \frac{T_{cm} - T_s}{Q/A} = R_0 + R_s. \tag{7}$$

The physical meaning of T_{cm} may be obtained by considering an idealized model for heat transfer in dropwise condensation. Referring to Fig. 2, we will divide the condensing surface to *inactive area* and *apparent effective area.* The first would represent the area covered by big drops with heat resistance through it, at least an order of magnitude higher than the average resistance. For example, if the average heat-transfer coefficient in a dropwise condensation experiment was 40 000 Btu/hft^2°F it can be shown, by neglecting the internal circulation, that all drops with radii bigger than 0·1 mm (for water) would have resistance through the liquid only (per unit area covered by the drop) about

10 times higher than the average resistance encountered in the experiment. For this reason the heat flux passing through such (inactive) drops will be neglected and the surface area covered by them will be considered to be the adiabatic area.



(a) Temperature distribution in the condensing plate



(b) Local surface-to-vapor resistance distribution

FIG. 2. Dropwise condensation model.
(a) Temperature distribution in the condensing plate.
(b) Local surface-to-vapour resistance distribution.

The rest of the area would be the apparent effective area over which microscopic active drops are distributed.

We realize now that the above model implies two types of constriction: macroscopic constriction due to the presence of relatively large inactive area and microscopic constriction due to the microscopic drops distribution over the apparent effective area. In order to take those two constrictions separately into account we express T_c as [see detail A in Fig. 2(a)]:

$$T_c \equiv \bar{T}_c + \Delta T_c'$$

and further using definition (4) write the expression for T_{cm} as:

$$T_{cm} = \frac{R_0}{A}\left[\int \frac{\bar{T}_c}{R}\,\mathrm{d}A + \int \frac{\Delta T_c'}{R}\,\mathrm{d}A\right] \equiv \bar{T}_{cm} + \Delta T_{cm}' \qquad (8)$$

Also from (6):

$$R_s = \frac{\bar{T}_{cm} - T_s}{Q/A} + \frac{\Delta T_{cm}'}{Q/A} \equiv R_{smac} + R_{smic}. \qquad (9)$$

The variation of $1/R$ over the apparent effective area is on a microscopic scale [see Fig. 2(b)]. The variation of $\bar{T}_c$, however, over the same area is on a macroscopic scale.

Assuming, as a part of the model, that the active drops are randomly distributed over the apparent effective area (consequently $1/R$ is also randomly distributed) and using the fact that the distribution of $\bar{T}_c$ is independent of the local microscopic variation of $1/R$ over the apparent effective area, we can evaluate $\bar{T}_{cm}$ as follows:

$$\bar{T}_{cm} = \frac{R_0}{A}\int_A \frac{\bar{T}_c}{R}\,\mathrm{d}A = \frac{R_0}{R_{eff}}\frac{1}{A}\int_{A_{eff}} \bar{T}_c\,\mathrm{d}A = \frac{1}{A_{eff}}\int_{A_{eff}} \bar{T}_c\,\mathrm{d}A. \qquad (10)$$

A_{eff} stands for the apparent effective area. Since it can be shown [14] that the mean value of $\Delta T_c'$ over the apparent effective area is zero, one can deduce from equation (10) that $\bar{T}_{cm}$ is the average temperature over A_{eff}. The macroscopic constriction resistance (R_{smac}), as defined by relation (9), could be calculated then from the known difference between mean surface temperature over the apparent effective area and the mean surface temperature over the whole surface (T_s). (See appendix A for a proof that T_s, the temperature one would obtain by extrapolating the linear temperature profile far from the surface, is also the mean surface temperature).

For the microscopic constriction, for each active drop constriction resistance is in series with resistance through liquid and liquid–

vapor interface. The resistance through liquid for any drop can be written in the form:

$$R_{\text{liquid}} \equiv \frac{(\Delta T)_{\text{drop}}}{Q_d} = \frac{C}{\pi k_l r} \tag{11}$$

where k_l is thermal conductivity of the liquid; Q_d is the heat rate through the drop; r is the radius of the drop and C is a constant. Different authors considered theoretically the problem of thermal resistance through a drop [7, 15, 16]. From their works, which neglect circulation of liquid inside drops, it follows that C must be less than $2/\pi$. From [7] it could be deduced that the value for C is approximately $\frac{1}{4}$. The microscopic constriction for a single active drop can be expressed [17] as:

$$\frac{\Delta T'_{cm}}{Q_d} = \frac{\psi}{4k_s r} \tag{12}$$

where:

$$\psi = (1 - \varepsilon^{0\cdot5})^{1\cdot5}. \tag{13}$$

ε is the fraction of the apparent effective area covered by the active drops and k_s is the thermal conductivity of the surface.

It follows now from equations (11) and (12) that for each active drop, regardless of its size, the ratio between the microscopic constriction and the resistance through the liquid respectively is of the order of k_l/k_s and hence, provided that the internal circulation inside drops is insignificant, the microscopic constriction effect on the dropwise condensation is negligible. Consequently, if the constriction resistance is an important factor in dropwise condensation, it would come, as it was proposed by Griffith [18], only through the macroscopic constriction term, and in this case, see Fig. 2(a), $\Delta T'_c$ is much smaller than $(\bar{T}_c - T_s)$.

2.2 *Evaluation of the constriction resistance*

It was concluded in the last section that the constriction resistance under condition of dropwise condensation can be expressed as:

$$R_s = R_{smac} + R_{smic} \simeq R_{smac} = \frac{\{1/A_{eff} \int_{A_{eff}} T_c \,\mathrm{d}A\} - T_s}{Q/A}. \tag{14}$$

Realizing that around each inactive drop will be an active area (partially covered by microscopic active drops randomly distributed) one can build a solution for an elemental heat channel with an adiabatic disc of radius c (comprising the area under the big drop) and an active area $\pi(b^2 - c^2)$ around it (Fig. 3). The (macroscopic) boundary condition over the active area was taken to be constant heat flux (justified by the randomness of size and position distribution of the active drops over the area). This model is treated mathematically in Appendix A. The solution for the resistance for one channel is given by equation (A.9). The expression for unit apparent area follows from (A.9) as:

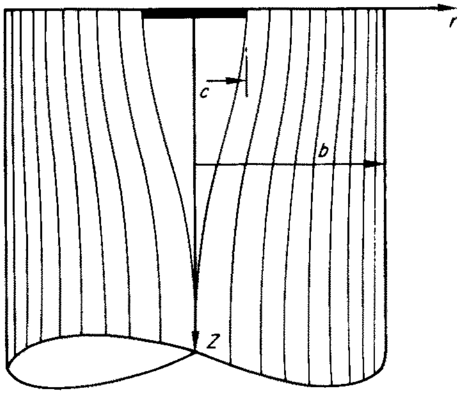


FIG. 3. Elemental heat channel.

$$R_s \simeq \frac{\beta^2}{(1 - \beta)^2} \frac{\psi'}{4k\sum c_i/A}. \tag{15}$$

c_i represents radii of inactive drops.

$\sum c_i/A$ can be either approximated as $(c \,.\, n)$ where c is the radius of a typical inactive drop and n is the number of inactive drops per unit area, or calculated from a picture showing the inactive drops distribution over the condensing surface, by passing an average (all representative) line across the surface: half the number of inactive drops crossed per unit length of the line would represent the required sum [17]. ψ', as given in (A.10) may be approximated by the relation:

$$\psi' \simeq \frac{32}{3\pi^2}(1 - \beta^{0.5})^{1.5}. \qquad (16)$$

We would like to point out here that formula (15), together with relation (16) gives lower limit for the constriction resistance since the simplification introduced in the development of expression (A.9), in the form given on the right-hand side of the equation, assumes that the ratio of the inactive area to the apparent area is the same for all elemental channels. The real distribution does deviate from this model. The consequence of this is that the actual constriction resistance would be somewhat higher than the value given by equation (15). It is of some interest to note that the introduction of a "typical inactive drop" concept, together with some algebraic simplification, would reduce (15) into:

$$R_s \simeq \frac{1}{3}\frac{\beta}{(1 - \beta)^{0.5}}\frac{c}{k}. \qquad (15a)$$

The above relation is convenient for a rough estimation of the constriction resistance when the exact droplets distribution is not available. By using the value for c, the typical radius of inactive drops, close to the radius of the maximum size droplets, and a reasonable estimation of β, relation (15a) should yield a fair estimate of R_s.

The assumption of equal area ratio (inactive area/active area) for each elemental heat channel, regardless of its size, could be avoided by considering effects of inactive drops of different size separately and adding all the effects. This approach which should yield the upper limit for the constriction resistance, is given in Appendix B.

The whole problem of the constriction resistance has been treated here as a steady state problem. This approach is justified by the fact that the time response required for R_s to attain most of its steady state value [19, 20] is much smaller than the average life of big drops, which are, as we already established, responsible for this type of the resistance.

In the preceding section, the constriction resistance in the dropwise condensation is related to the geometry (distribution) of inactive drops. Here we imply the existence of a distribution of droplets which is stationary (in the statistical context of the word) and hence, capable of describing a steady state dropwise condensation (over sufficiently large area of the condensing surface), in other words, we use the fact that there exists a distribution which represents average conditions over condensing surface. This distribution of course, depends on most of the numerous factors which constitute a whole mechanism of dropwise condensation (see, for example, [9]). The analysis of those factors is beyond the scope of this work.

2.3 *Quantitative evaluation of constriction resistance from available data*

Some experimental data do contain sufficient information for quantative estimation of constriction resistance in the solid. Griffith and Lee [13] reported heat transfer coefficient measured for the dropwise condensation of steam at 1 atm on a horizontal surface facing down. The promoter was oleic acid. The surface materials used were copper, zinc and stainless steel, all with mirror finished gold-plated surfaces. The steam velocity was 27 ft/s. For copper, with thermal conductivity $k = 220$, the observed value for the heat-transfer coefficient was:

$$h_{\mathrm{I}} = 10000\ \mathrm{Btu/hft^2\,°F}$$

for stainless steel, $k = 10$:

$$h_{\mathrm{II}} = 2000\ \mathrm{Btu/hft^2\,°F}.$$

The heat transfer coefficients in both cases were independent of ΔT in the range of temperature difference variation of 2–20°F.

The heat-transfer coefficient is given by equation (7). For a fixed droplets distribution, R_0, equation (3), would be the same (independent of the surface properties). On the other hand R_s see equation (15), for a fixed distribution is inversely proportional to the surface thermal conductivity. So:

$$\frac{(R_s)_{\mathrm{I}}}{(R_s)_{\mathrm{II}}} = \frac{k_{\mathrm{II}}}{k_{\mathrm{I}}}. \tag{17}$$

Since h was independent of ΔT in their experiments one can assume that in the whole range of ΔT tested there was at least one case of statistically identical droplets distribution on the two different surfaces tested. Considering the the copper surface and the stainless steel surface under stated conditions: ($R_{0\mathrm{I}} = R_{0\mathrm{II}}$) one can write two equations of type (7) for the two respective surfaces. Then together with (17) the values for $R_{s\mathrm{I}}$, $R_{s\mathrm{II}}$ and R_0 can be calculated. In this manner it was found that resistance in the stainless steel surface accounted for about 84 per cent of the total resistance, ($R_{s\mathrm{II}}^{-1}$ = 2380 Btu/hft^2°F) and in the case of the copper surface 19 per cent ($R_{s\mathrm{I}}^{-1}$ = 52400 Btu/hft^2°F). The resistance through droplets and liquid vapor interface was in both cases R_0^{-1} = 12350 Btu/hft^2°F per unit area of the condensing surface.

The high value for R_0 (as well as the total resistance) encountered in the experiments discussed, suggests the presence of non-condensible gases in the system. If so, the relative effect of constriction resistance calculated above is less than it would be under conditions of absence of non-condensibles (for the fixed geometry). Also, the gold-plating on both surfaces acted in the direction of reducing the differences of the constriction resistance, on the tested surfaces. It follows then that the above quantitative evaluation gave the lower limit for the constriction resistance effects for the droplets distribution considered.

The other set of experiments in dropwise condensation made on the stainless steel and copper surfaces, respectively, is reported by Tanner *et al.* [12]. The surface was vertical; the promoter was montanic acid. The heat-transfer coefficient, for stainless steel as the condensing surface, was (approximately) in the range of 6000–8000 Btu/hft^2°F (higher values for higher heat fluxes). In cases of copper as the condensing surface h = 42000 Btu/hft^2°F. Using the same calculation procedure as in the previous case, and assuming that the same heat-transfer geometry will occur for the same heat flux, it was calculated that (in average) for stainless steel surface constriction resistance was 85 per cent of the total resistance: $(R_s)^{-1}_{st.s.}$ = 8800 Btu/hft^2°F; for copper surface the contribution of the resistance occurring in the metal was 20 per cent: $(R_s)^{-1}_{copp}$ = 210000 Btu/h/ft^2°F. We should say, however, that validity of our numerical evaluation in the last case is subject to acceptability of the assumption that the same preparation and the same promoter had the same effect on the two different surfaces. On the other hand, in view of the results from [13] there should not be much doubt that the difference in the heat-transfer coefficient for the two materials tested in [12] came primarily due to the difference in respective thermal conductivities. Accepting then that the above evaluation cannot be much off, we should like at this stage to comment on some difference between the two cases discussed. It could be noted that in the case of vertical position of the condensing surfaces, constriction resistance was much lower. That should be expected, since, for a given surface material the constriction resistance depends only on the distribution of large (inactive) drops on the condensing surface and the latter strongly depends on the surface position (in the presence of gravity). The heat transfer coefficient was also much higher for the vertical surfaces. That came not only due to the decrease in the constriction resistance but also due to a substantial decrease in R_0 in the experiments done by Tanner *et al.*, which was lower mainly because of the absence of non-condensibles in the condensing system. The fact that relative contribution of constriction resistances for the same material calculated from experimental results of Griffith and Lee, and Tanner *et al.*, is the same, is obviously accidental, and no generalisation should be made based on the fact that for the stainless steel surfaces R_s was about 84 per cent of the total resistance and for copper about 20 per cent for both sets of experiments.

The important conclusions that can be drawn from the above evaluations are:

(i) The constriction resistance is present in dropwise condensation.
(ii) Its contribution to the overall resistance could be significant and therefore cannot be left out from any dropwise condensation model.

3. OVERALL HEAT-TRANSFER COEFFICIENT

For an active drop of hemispherical shape, of radius r, with the average temperature of the base (solid–liquid interface) T_c, and heat-transfer coefficient h_e between the liquid and vapor (at T_v), Umur and Griffith [7] derived the following expression for the heat-transfer rate through the drop:

$$\frac{Q_d}{rk_l(T_v - T_c)} = 2\pi \sum_{n=0}^{\infty} \frac{(2n+1)(4n+3)}{1 + \frac{k_l}{h_e r}(2n+1)} \times \left[\int_0^1 P_{2n+1}(x)\,\mathrm{d}x\right]^2. \quad (18)$$

Here P_n are Legendre polynomials. Equation (18) is presented graphically in Fig. 4. If one formally considers that the flow of heat from vapor to solid is controlled by two resistances, namely the interfacial resistance ($\frac{1}{2}\pi h_e r^2$) and the conduction resistance through the drop ($\frac{1}{4}\pi k_l r$), one can write for the heat-transfer rate the following expression

$$\frac{Q_d}{rk_l(T_v - T_c)} = \frac{4\pi}{1 + (2k_l/h_e r)}. \quad (19)$$

Factor $\frac{1}{4}$ in the expression for the resistance through drop was chosen in order that relation (19) yield good agreement with the exact solution of the problem, equation (18). As it can be seen from Fig. 4, the predictions for the heat-transfer rate through the drop from the two equations are almost identical.

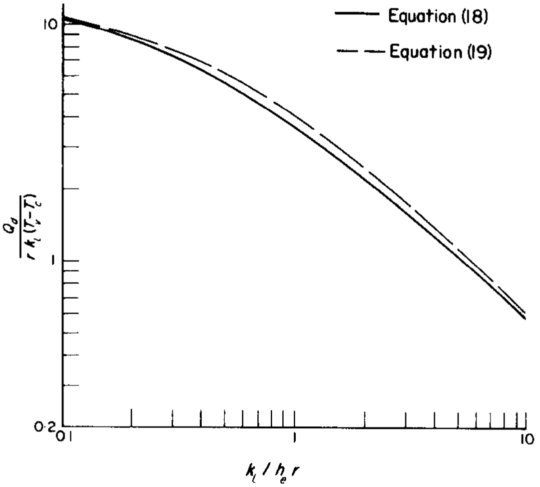


FIG. 4. Variation of heat transfer through the drop with $k_l/h_e r$.

The interfacial heat-transfer coefficient can be estimated from the following formula, developed in [7]:

$$h_e = \left(\frac{2\gamma}{2-\gamma}\right)\left(\frac{g}{2\pi}\right)^{\frac{1}{2}} \frac{h_{fg} P_{sat}}{R^{\frac{1}{2}} T_v^{\frac{1}{2}}} \left(1 - \frac{r^*}{r}\right).$$

The quantities in the above equation are introduced in the nomenclature. From (19), (3) and (5) follows the expression for the heat flux per unit area as:

$$\frac{Q/A}{T_v - T_{cm}} = 4\pi k_l \sum_j \left(\frac{r_j}{1 + (2k_l/h_e r_j)}\right)/A. \quad (20)$$

The expression for the heat transfer coefficient can be written now from equations (7), (10), (15), (16) and (20) as:

$$h^{-1} = \frac{1}{4\pi k_l}\left[\sum_j \left(\frac{r_j}{1 + (2k_l/h_e r_j)}\right)/A\right]^{-1} + \frac{\beta^2}{(1-\beta)^2} \frac{(1 - \beta^{0 \cdot 5})^{1 \cdot 5}}{3 \cdot 7 k_s} \left[\left(\sum_i c_i\right)/A\right]^{-1}. \quad (21)$$

Equation (21), at least in this form, is not intended for a practical calculation of the heat-transfer coefficient. The main purpose of having it here is to emphasize the effects of microscopic and macroscopic heat-transfer geometry on dropwise condensation. In expression (21) the two effects are given explicitly through the two separate terms.

The summation in the first term of the above equation need not be terminated at a given drop size, i.e. the contribution to the heat transfer of the middle size and larger size drops could be taken, principally, into account by extending summation of the first term over the whole droplets population. Hence, the model does not neglect the contribution to the heat transfer by drops bigger than certain size; what it does neglect, however, is the effect of a very low heat flux, compared to the assumed no heat flux, at the base of big drops, on the temperature distribution in the vicinity of such drops (the constriction resistance model).

The validity of (21) is limited by the following major assumptions: absence of non-condensibles, negligible effect of liquid circulation inside drops and insignificant temperature drop across liquid–vapor interface due to the surface tension effect.

4. DISCUSSION

In general, all main factors which govern the mechanism of dropwise condensation could be roughly divided into two groups, mainly, those which primarily affect directly the droplets distribution (nucleation, coalescence, surface inclination, vibration, pressure, etc.) and those which determine the heat-transfer coefficient for a *given distribution* (conduction through drops, constriction phenomena in the plate and the liquid–vapor interface heat-transfer coefficient (h_e), with the factors which determine it). Of course, there are some cross effects between factors of the groups as, for example, system pressure which affects both the nucleation density in the first group and h_e in the second one. Nevertheless, in spite of some obvious disadvantages, the above concept has some merits, and in this work it was implicitly adopted. We limited the scope of the work presented here only to consideration of the factors in the second group (by starting with a droplets distribution). There, too, we considered in detail only the constriction phenomena; the others are brought into consideration only to show relative significance of the constriction.

It was established how the thermal conductivity of the condensing surface affects the heat transfer in dropwise condensation. It was shown that the available temperature potential for the active droplets growth ($T_v - T_c$), is less than ΔT ($\equiv T_v - T_s$). The average value of available temperature difference, ΔT_{eff} ($\equiv T_v - T_{cm}$), for the active droplets growth, can be expressed, using equation (7), as follows:

$$\Delta T_{eff} = \Delta T(1 - hR_s).$$

The product $h \cdot R_s$ represents the fraction of the total resistance which is caused by the constriction phenomena in the plate. It was also shown how both h and R_s depend on the thermal conductivity of the condensing surface as well as the droplets distribution. Hence, as a consequence, one should expect that a rate of growth of an active drop depends also on those two factors.

Vast experimental data showed (see for example [9]) that growth rate could be expressed as $dr^2/dt = \text{const.}$ (One could get the same expression from equation (19) for conditions where $(2k_l/rh_e) \ll 1$). The constant in the expression for the growth rate is a function of the fluid properties and ΔT_{eff} (directly proportional to ΔT_{eff}). The use of ΔT instead of ΔT_{eff} does not allow for changes in the growth rate due to the variations of any of the parameters which do affect the droplets distribution as well as due to the changes in the surface conductivity.

In addition to the average effect of the distribution on the growth rate (through ΔT_{eff}), there could be strong local effects on the growth rate due to local changes in T_c. For example, if a closely packed group of active drops is present on a part of the condensing surface, the heat flux over that part of the surface would be relatively high and hence, using our initial argument (see Fig. 1), T_c would be relatively high; consequently, the available temperature difference, $T_v - T_c$, would be low, causing a lower growth rate. So, clearly, the growth rate must be a function of the local density of the active drops. Indeed, the experimental results of McCormick and Westwater [9] have demonstrated conclusively that the growth rate goes down as the average distance between drops decreases (for otherwise the same condition). We do believe that their results can be attributed, at least partially, to the above discussed effects. Westwater [21] indicated the possible role of the constriction phenomena in the results of [9] by stating that "conduction through the metal substrate probably is influenced also by the spacing of drops".

There are certainly other phenomena in dropwise condensation which could be entirely or partially attributed to the effects of the constriction resistance. A quantitative evaluation of such effect presently cannot be given, since the most reported experimental data on dropwise condensation are performed on the copper surfaces.

We would like here to emphasize the significance of having experimental data generated on the condensing surfaces with different thermal conductivities. From the results, we would get an answer to a particular question: what is the heat transfer in dropwise condensation for a specified surface material, or more complicated one: what changes in the heat transfer would one get with variations of one of the governing parameters (again for the specified surface material). In addition to the above, we would get, by comparing results performed on surfaces with different thermal conductivities, (i) what is the significance of the constriction resistance for each surface and (ii) what part of the overall resistance or, equivalently, distribution, were mostly affected by changes of a specified parameter.

5. CONCLUSION

(1) From theoretical consideration it was shown that the constriction resistance caused by deviations of heat flow lines in the solid is always present in dropwise condensation.

(2) From available experimental data [19, 13] it was deduced that the constriction resistance is significant in this type of heat transfer: about 84 per cent of the total resistance was the contribution of the constriction resistance in the case of stainless steel as the condensing surface; for copper surfaces the contribution was about 20 per cent. Those figures reflect only specific conditions of the experiments considered.

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APPENDIX A

Constriction Resistance Model—Elemental Heat Channel

To each inactive drop will belong certain amount of apparent active area covered randomly by microscopic (active) drops. In this fashion one can divide the whole condensing surface in N parts, where N is the number of inactive drops. Approximating the apparent active area belonging to a big drop with an annulus around the drop, one can reduce the whole problem to the problem of an elemental heat channel given in Fig. 3. The area π_c^2 represents the area under the big drop. By definition of the elemental heat channel, the cylinder $r = b$ is the adiabatic cylinder.

The problem is defined with:

$$\nabla^2 T = 0 \tag{A.1}$$

$$\left.\begin{aligned} -k\left(\frac{\partial T}{\partial z}\right) &= \frac{Q}{\pi(b^2-c^2)} \qquad c < r < b \\ -k\left(\frac{\partial T}{\partial z}\right) &= 0 \qquad o < r < c \end{aligned}\right\} z = 0 \tag{A.2}$$

$$-k\left(\frac{\partial T}{\partial z}\right) = \frac{Q}{\pi b^2} \qquad z = \infty \tag{A.3}$$

$$-k\left(\frac{\partial T}{\partial r}\right) = 0 \qquad r = b. \tag{A.4}$$

The solution to equation (A.1) which satisfied (A.3) and (A.4) is:

$$T = C_0 - \frac{Q}{K\pi b^2} z + \sum C_n e^{-\alpha_n z} J_0(\alpha_n r) \tag{A.5}$$

where α_n are equivalues determinable from the following relation:

$$J_1(\alpha_n b) = 0. \tag{A.6}$$

From (A.2):

$$C_n = -\frac{2Qc}{\pi k(b^2-c^2)}\frac{J_1(\alpha_n c)}{(\alpha_n b)^2 J_0(\alpha_n b)}. \tag{A.7}$$

The temperature distribution now can be written as:

$$T = C_0 - \frac{Q}{k\pi b^2} z - \frac{2Qc}{\pi k(b^2-c^2)} \sum_{n=1}^{\infty} e^{-\alpha_n z} \frac{J_1(\alpha_n c)\, J_0(\alpha_n r)}{(\alpha_n b)^2\, J_0^2(\alpha_n b)}. \tag{A.8}$$

It follows from (A.8) that C_0 is T_s, the temperature which could be obtained by extrapolation of the temperature profile far from the interface. By virtue of (A.6) it is also the mean temperature over $z = 0$ surface.

Defining the constriction resistance for the cylinder as:

$$R_{cyl} \equiv \frac{T_{cm} - T_s}{Q}$$

and taking T_{cm} as the average temperature over the active area $\pi(b^2 - c^2)$ one easily obtains for the resistance:

$$R_{cyl} = \frac{c^3}{4k(b^2-c^2)^2}\psi \simeq \frac{\beta^2}{(1-\beta)^2}\frac{1}{4kc}\psi \tag{A.9}$$

where-

$$\psi \equiv \frac{16}{\pi}\frac{b}{c}\sum_{n=1}^{\infty}\frac{J_1^2(\alpha_n c)}{(\alpha_n b)^3 J_0^2(\alpha_n b)} \simeq \frac{32}{3\pi^2}\left(1-\frac{c}{b}\right)^{1\cdot 5} \tag{A.10}$$

and β is the fraction of the area covered by big drops.

APPENDIX B

A Model for Constriction Resistance for Multiple Heat Channels

Let us assume for a moment that all inactive drops presented at a given moment on the condensing surface could be divided according to their size, in groups. Let D_i and n_i be the diameter of drop size, and the number of drops per unit area, respectively, which belong to i-th group.

Consider the group with the largest diameter (D_1, n_1). Suppose that only this group of inactive drops is present on the surface and causing the constriction resistance to heat flow from the surface. For each elemental heat channel associated to individual members of the group we can write [from equation (49)]

$$\frac{(\Delta T)_1}{(Q/A)/n_1} = \frac{8(c_1/b)^4}{3\pi^2 k[1-(c_1/b_1)^2]^2} \times \left(1-\frac{c_1}{b_1}\right)^{1\cdot 5}\frac{1}{c_1}. \tag{B.1}$$

$(\Delta T)_1$ is the temperature drop associated with the resistance imposed by drops of group 1. We can further write

$$\pi c_1^2 \cdot n_1 = \frac{\text{active area}}{\text{apparent area}} = \frac{c_1^2}{b_1^2}$$

or

$$\frac{\pi D_1^2 n_1}{4} = \left(\frac{c_1}{b_1}\right)^2 = \beta_1.$$

From (B.1), one can express $(\Delta T)_1$ as follows:

$$(\Delta T)_1 = Q/A \cdot \frac{16}{3\pi^2 k}\frac{\beta_1^2(1-\beta_1^{0\cdot 5})^{1\cdot 5}}{(1-\beta_1)^2 n_1 D_1}. \tag{B.2}$$

Consider now the drops of group 2 distributed over the area not covered by drops bigger than D_2 (in this case, the area not covered by drops of group 1).

For this group:

$$\frac{\pi c_2^2 n_2}{1-f_2} = \frac{\text{active area}}{\text{apparent available area}} = \frac{c_2^2}{b_2^2} \equiv \beta_2$$

where f_2 is the fraction of the area covered by drops of group 1.

The temperature drop associated to the constriction resistance caused by drop of group 2 can be written in the same form as equation (B.2) (replacing subscript 1 with 2). Similarly, for i-th group, the temperature drop has the same form as (B.2), where

$$\beta_i \equiv \frac{\pi D_i^2 n_i}{4(1-f_i)}.$$

f_i is the fraction of the area covered by drops bigger than D_i.

Summing (ΔT) for all the groups of inactive drops, and dividing by Q/A, one gets the expression for the constriction resistance, calculated by this model in the following form

$$R_s = \frac{16}{3\pi^2 k}\sum_i \frac{\beta_i^2(1-\beta_i^{0\cdot 5})^{1\cdot 5}}{n_i D_i(1-\beta_i)^2}. \tag{B.3}$$

β_i represents the ratio between the area covered by the drops of group i and the apparent available area; this ratio could be made as small as desired by deriding the whole population of inactive drops in sufficient number of groups. Hence, making β_i sufficiently small so that $\beta^{0\cdot 5} \ll 1$, we can simplify (B.3) into

$$R_s \simeq \frac{1}{3\pi k}\sum_i \frac{n_i D_i^3}{(1-f_i)^2},$$

or for a continuous drop distribution

$$R_s = \frac{1}{3\pi k}\int_{D_{\min}}^{D_{\max}} \frac{N(D)D^3\,\mathrm{d}D}{[1 - f(D)]^2} \qquad \text{(B.4)}$$

$N(D)\,\mathrm{d}D$ represents the number of drops per unit area of diameter size between D and $D + \mathrm{d}D$. $f(D)$ is the fraction area covered by drops greater than D. $D_{\min}$ and $D_{\max}$ are the diameters of the smallest and the largest inactive drops on the condensing surface, respectively.

This method will yield somewhat higher values for R_s since the separate accounting of all groups assumes that the constriction of heat flow lines due to a group of drops of a given size is completely accomplished before deviation due to the next group (of drops of next smaller size) starts to develop. The real constriction would depart from this model, producing, as a consequence, a lower overall resistance.

Résumé—L'effet des propriétés thermiques superficielles sur la condensation en gouttelettes est examiné. L'explication physique pour cet effet est donnée en considérant les phénomènes de contraction dans les surfaces métalliques provoqués par une distribution non-uniforme de flux de chaleur sur les surfaces de condensation.

On montre à partir des données disponibles que les propriétés thermiques superficielles sont un des facteurs principaux dans la condensation par goutelettes. On trouve que, pour une surface de condensation en acier inoxydable, la résistance qui existe dans la plaque elle-même due à la non-uniformité du flux de chaleur sur la surface, contribuait à environ 84 pour cent de la résistance totale; pour le cuivre, cette contribution était de 20 pour cent.

Zusammenfassung—Der Einfluss von thermischen Materialkonstanten der Kondensationswand wird untersucht. Die physikalische Erklärung für diesen Einfluss ist die Hemmung des Wärmeflusses durch die ungleichmässig verteilte Wärmestromdichte in der Wand. Aus bekannten Versuchsergebnissen wird geschlossen, dass die thermischen Materialkonstanten die Geschwindigkeit der Tropfenkondensation mitbestimmen.

Es wird ermittelt, dass für eine Kondensationswand aus Chrom-Nickel-Stahl 84 Prozent des gesamten Wärmewiderstandes auf die Behinderung des Wärmeflusses aufgrund der ungleichmässigen Wärmestromdichte entfällt; bei Kupfer beträgt dieser Anteil am gesamten Wärmewiderstand 20 Prozent.

Аннотация—Рассматривается влияние теплофизических характеристик поверхности на капельную конденсацию. Физическое толькование этого эффекта заключается в том, что неравномерное распределение теплового потока по поверхности конднесации приводит к сокращению отдельных участков поверхности металла. С помощью полученных данных показано, что теплофизические свойства поверхности являются одним из определяющих факторов при капельной конденсации. Установлено, что при поверхности конденсации из нержавеющей стали около 84% общего сопротивления в самой пластине составляет сопротивление за счет неравномерности теплового потока на поверхности; для меди это сопротивление составляло 20%.